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The objective of the research supported by AFOSR was development of new techniques for rigorous and efficient computer-aided analysis and design of essentially nonlinear systems. Applications of these techniques will include stability analysis of fighter aircraft control algorithms in the case of gain scheduling and actuator rate and amplitude saturation. A number of useful analysis and design tools was developed with the support of the AFOSR grant. A new framework was proposed for the problem of auto-oscillations in-duced by a relay feedback. For the problem of stabilization of linear time-invariant sys-tems with bounded control, a simple method of designing a stabilizing output feedback is proposed, based on "scheduling" a single-parameter family of controllers, so that the higher gain controllers are used closer to the origin. A number of contributions to the method of Integral Quadratic Constraints (IQC) has been made, enhancing the handling of important nonlinear blocks, such as saturation, relay, and hysteresis. In particular, a new set of IQC is derived for a class of "rate limiters", modelled as a series connections of saturation-like memoryless nonlinearities followed by integrators. In addition, more flexible version of the general IQC analysis framework is presented, and the IQC analysis software also made a significant step forward after the package iqc-beta" was introduced. The issue of designing aggressive globally stabilizing controllers in some benchmark me-chanical systems was addressed, and a methodology for robustness analysis of forced and self-induced oscillations in nonlinear systems has been developed.

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Analysis of Essentially Nonlinear Systems

by Alexandre Megretski

The objective of the research supported by AFOSR was development of new techniques for rigorous and efficient computer-aided analysis and design of essentially nonlinear systems. Applications of these techniques will include stability analysis of fighter aircraft control algorithms in the case of gain scheduling and actuator rate and amplitude saturation.

A number of useful analysis and design tools was developed with the support of the AFOSR grant. A new framework was proposed for the problem of auto-oscillations induced by a relay feedback. For the problem of stabilization of linear time-invariant systems with bounded control, a simple method of designing a stabilizing output feedback is proposed, based on "scheduling" a single-parameter family of controllers, so that the higher gain controllers are used closer to the origin. A number of contributions to the method of Integral Quadratic Constraints (IQC) has been made, enhancing the handling of important nonlinear blocks, such as saturation, relay, and hysteresis. In particular, a new set of IQC is derived for a class of "rate limiters", modelled as a series connections of saturation-like memoryless nonlinearities followed by integrators. In addition, more flexible version of the general IQC analysis framework is presented, and the IQC analysis software also made a significant step forward after the package "iqc-beta" was introduced. The issue of designing agressive globally stabilizing controllers in some benchmark mechanical systems was addressed, and a methodology for robustness analysis of forced and self-induced oscillations in nonlinear systems has been developed.

With the support from the AFOSR under the grant "Analysis of Essentially Nonlinear Systems", the following progress has been made in 1996-1999.

Technology transfer

Below, the project "Robust Stability Analysis of the Longitudinal Control System of a Tail-less Aircraft", developed in cooperation and with partial support from the Boeing Inc., is discussed.

An aircraft is said to be operated in a *trim* condition when the body axis accelerations are zero. In other words, no resultant forces or torque act on the aircraft. The purpose of the longitudinal controller of an aircraft is to help pilots to move the aircraft from one trim condition to another, or to stabilize the aircraft in an operating trim condition while the aircraft is subjected to disturbances. The dynamics of the aircraft is highly nonlinearly depending on the trim condition at which the aircraft is operated [16]. For simplifying the controller design task, the control scheduling methodology is usually adopted. In control scheduling, a set of controllers are designed to stabilize the linearized aircraft models around each trim conditions. When the aircraft moves from one trim condition to another, the controllers switch correspondingly. Therefore, instead of a complex nonlinear control design problem, one only has to design controllers for a set of linear time invariant systems which makes the control design task dramatically simplified. However, in exchange, the stability analysis of the control system becomes important issue, because there is a need to guarantee stability when the system is switching between trims conditions.

We start with a longitudinal control system of a tailless aircraft for the aircraft operated at Mach number 0.9 between the angle of attack of 1.65 degrees to 7 degrees. The controller and the plant equations are given in the state space form with 2 states, 3 inputs, and 4 outputs of the plant, while the controller has 18 states, 5 inputs and 3 outputs. There are three actuators in this control system. The rate limiter blocks between the controller and the aircraft plant are to model the limited rate of variation of these actuators. The rate limiter is the interlink of a saturation nonlinearity and an integrator with negative feedback. Thus, for robustness analysis, the system can be modeled as having nonlinear/uncertain blocks of three types:

- saturation nonlinearity (to model the rate limiter)

- time-varying gain (to model gain scheduling effects)

- unmodeled time-varying dynamics (to model errors in the first two blocks)

The purpose of the study was to verify the stability of the control system when the aircraft is operated in the continuous change of angle of attack within 1.65 to 7 degrees.

Details of the research results can be found on the Web at

http://www.mit.edu/people/ameg/report.ps.gz

Below, some major findings are reported.

Case 1: the dynamics of the actuators are not modeled. In this case, stability of the control system can be verified for arbitrary rate of variation of $|\dot\alpha|$.

Case 2: one of the actuators is rate limited. When one of the actuators is rate limited, the control system is stable for the angle of attack α varying arbitrarily fast. However, it does not appear that this can be proven using the classical IQC listed in [1]. Only after adding the newly reported IQC from [15] was it possible to derive a stability result. The same remark is true for the next two findings.

Case 3: first and second actuator are rate limited. Stability of the control system can still be verified for arbitratily fast variation of angle of attack.

Case 4: second and third actuator are rate limited. It was not possible to prove robustness with respect to arbitrarily fast variations of the angle of attack. However, with the addition of the IQC for slowly time varying gain [1], stability of the control system was verified for $|\dot\alpha| \le 10$.

Case 5: first and third actuator, or all of the actuator have limited rate of variation. If the first and the third actuator, or all of the actuators are modeled as rate limiters, after transforming the system to standard robust stability analysis setup, the LTI nominal parts of the system in these two cases have unstable poles. Therefore, the IQC approach stability analysis is not directly applicable. Some extra efforts should be done before the analysis can be performed. To be more specific, first we have to assume the states of the system are within certain range and do loop transformation to make the nominal system become a stable one. After the transformation, we have a stable nominal system linked with a new uncertain operator. Then we derive the IQCs for the new operator and do stability analysis. Suppose we get a positive result, which means there is a multiplier which can be used to prove the system is stabe, provided all the states of the system are within the preassumed range. Then based on the multiplier, a Lyapunov function can be derived for verifying whether the states are really within the range we assumed. If they are, a positive stability result is obtained. If they are not, then reassume a new range and do the whole analysis again. The existence of the corresponding (non-quadratic) Lyapunov functions is guaranteed by the theory. However, the practical computation method of the Lyapunov functions in the IQC analysis is a subject of future research.

IQCβ: Software for IQC framework robust stability analysis

IQCβ is a collection of MATLAB functions designed to streamline rigorous computer-aided analysis of stability and performance of nonlinear, uncertain, and time-varying dynamical systems. Both the theoretical foundation and the language of the toolbox are

based on the method of Integral Quadratic Constraints (IQC). The software provides an environment to users, such that users can easily handle the signals, parameters, frequency-dependent quadratic forms, and other stuffs in IQC theory as MATLAB variables. It also provides a library of IQC descriptions of many typical blocks, such as saturation, unmodeled LTI dynamics, etc. IQCβ also allows to build SIMULINK diagrams by using a set of special IQC icons, which can consequently be analyzed for stability and robustness.

IQCβ is written by a group of researchers at M.I.T. of US and Lund Institute of Technology of Sweden. Free copy of this software and can be found on the webpage: http://www.mit.edu/people/ameg/home.html.

IQCβ provides a graphical user interface based on SIMULINK 2, which allows users describe their systems in terms of block diagrams. First users draw the block diagrams in SIMULINK environment. The nominal part of the system is built by standard SIMULINK blocks, while for the uncertain part of the system, users put the "IQC" blocks provided by IQCβ. Every IQC block points to a MATLAB M-file, in which the integral quadratic constraints of the input and output signals of the block are described in the language of IQCβ. Then use a program called "`iqc_gui.m`", which serves as a translator from GUI to language of IQCβ, to translate the block graphics to IQCβ description and solve the corresponding the optimization problem for stability analysis.

Figure 1 shows the block diagram description of the aircraft control system from the previous subsection. The IQC blocks used in this analysis as well as the M-files they point to are listed in the following.

Relay feedback systems

In the paper [2], a new framework is proposed for the use of the standard robust control technique in the analysis of highly nonlinear dynamics. For the problem of studying auto-oscillations induced by a relay feedback, a modification of the method of Integral Quadratic Constraints is developed. In particular, it prove that sufficiently small (in the l^1 norm) higher-order linear time-invariant perturbations of a second-order system with a relay feedback preserve the unique globally attractive limit cycle.

Gain scheduling for saturated control systems

In [6], for the problem of stabilization of linear time-invariant systems with bounded control, a simple method of designing a stabilizing output feedback is proposed. The approach is based on "scheduling" a single-parameter family of controllers, so that the higher gain controllers are used closer to the origin. The resulting controller is capable of producing the "agressive" control, which stays at the control limit most of the time, can be applied to a non-symmetric control limits, and has rigorously proven performance assurances. It is shown that, in case the open-loop plant is not exponentially unstable, the closed-loop map from the plant disturbances and sensor noise to the state and control variables is L_2-bounded. Moreover, if the open loop plant is exponentially unstable, and has a pole at zero, non-zero sensor noise does not de-stabilize the plant.
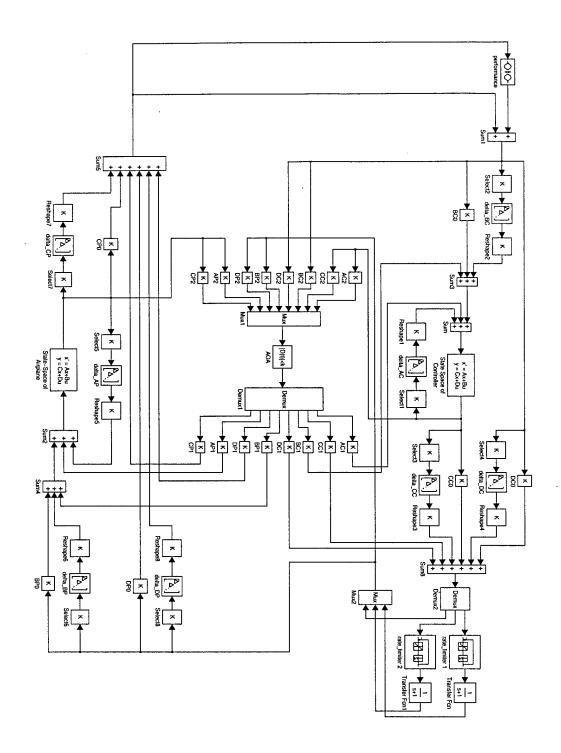
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Figure 1: Block Diagram of the Control System

Nonlinear vs. linear control

A class of examples of a robust control design problem has been discovered in [7, 8], with a linear parameter-varying system which cannot be stabilized by a linear feedback, while it can be stabilized easily using a nonlinear feedback.

New IQC for quasi-concave nonlinearities

In the paper [15], a new set of Integral Quadratic Constraints (IQC) is derived for a class of "rate limiters", modelled as a series connections of saturation-like memoryless nonlinearities followed by integrators. The result, when used within the standard IQC framework, is expected to be widely useful in nonlinear system analysis. For example, it enables "discrimination" between "saturation-like" and "deadzone-like" nonlinearities and can be used to prove stability of systems with saturation in cases when replacing the saturation block by another memoryless nonlinearity with equivalent slope restrictions makes the whole system unstable. In particular, it is shown that the L_2 gain of a unity feedback system with a rate limiter in the forward loop cannot exceed $\sqrt{2}$.

Extending the IQC framework

In a series of papers [9, 10, 11, 12, 15], a number of contributions to the method of IQC has been made, enhancing the handling of important nonlinear blocks, such as saturation, relay, and hysteresis. In addition, more flexible version of the general IQC analysis framework is presented, which relaxes the homotopy and boundedness conditions, and is more aligned with the language of the emerging IQC software. The IQC analysis software also made a significant step forward after the package "iqcβ" was introduced. "iqcβ", available from

`http://web.mit.edu/ameg/www/home.html`

allows its user to handle easily a variety of problems of robustness analysis.

Agressive control in nonlinear systems

The issue of designing agressive globally stabilizing controllers in some benchmark mechanical systems was addresss in [13, 14]. In [13], for a class of underactuated nonlinear systems, a global change of coordinates to transform the dynamics of the system into a desired form which consists of a lower order nonlinear subsystem plus a chain of integrators was discovered, intristically related to the use of impulsive controls. Then, a control Lyapunov function and associated control law for the lower order subsystem is designed. Finally, a backstepping procedure is used to derive the control Lyapunov function and the controller for the whole system. The obtained controller renders the origin semiglobally asymptotically stable. As an special case, this procedure is applied to the Acrobot example which is a two-link planar robot with a single actuator at the elbow.

Robustness of periodic orbits

In [4, 5], a methodology for robustness analysis of forced and self-induced oscillations in nonlinear systems is being developed. The linearization of the dynamics around a periodic solution to an autonomous ODE has a neutrally stable mode. It is therefore not suitable to directly apply standard techniques for robustness analysis of time-varying systems. It is shown that the system can be transformed in such a way that the neutral mode can be removed from the stability analysis. Any method for robust stability analysis of periodically-varying linear systems can now be applied.

Analysis of Poincare maps

The paper [3] presents sufficient conditions for regions of attraction of limit cycles in relay feedback systems. Conditions, in the form of Linear Matrix Inequalities (LMIs), are presented that guarantee the stability of a limit cycle in a reasonably large set around it. These results differ from previous local results as they take into account the higher order terms of the Poincare map nonlinearity.

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